On-Policy vs Off-Policy Algorithms

CMPT 729 G100

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Overview

- On-Policy vs Off-Policy
- On-Policy Algorithms
- Off-Policy Algorithms
- Trade-Offs

On-Policy vs Off-Policy

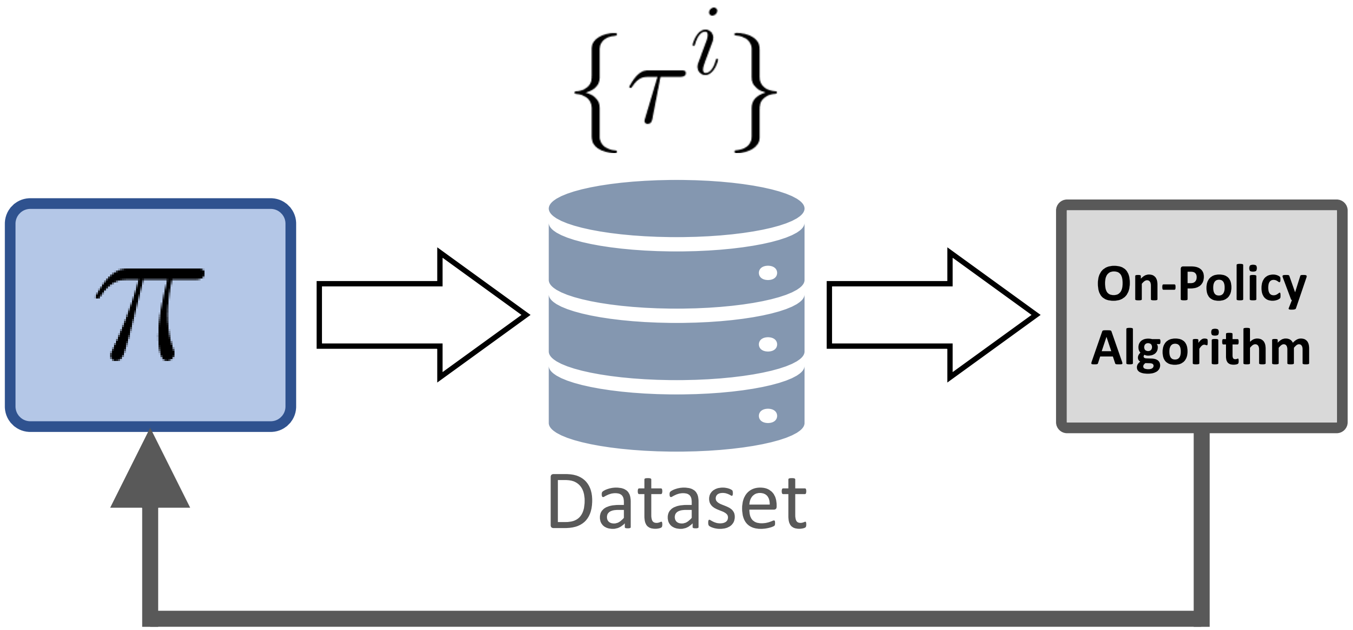
On-policy:

- Model can be update using *only* data collected with the model

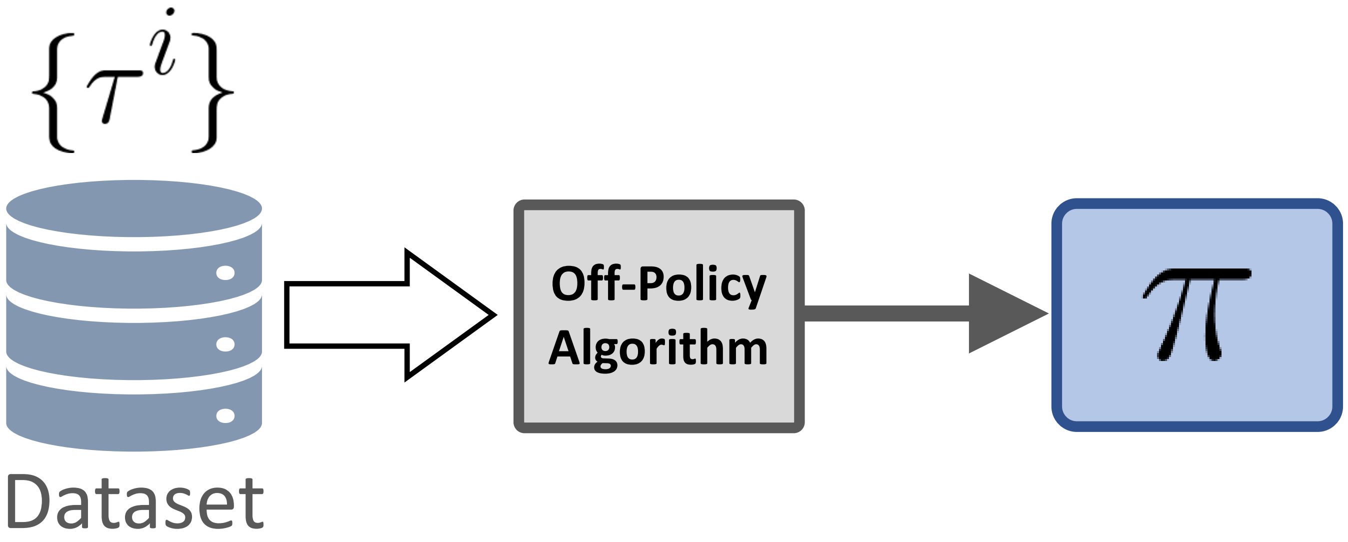
Off-policy:

- Model can be updated using data collected from *other* sources

On-Policy

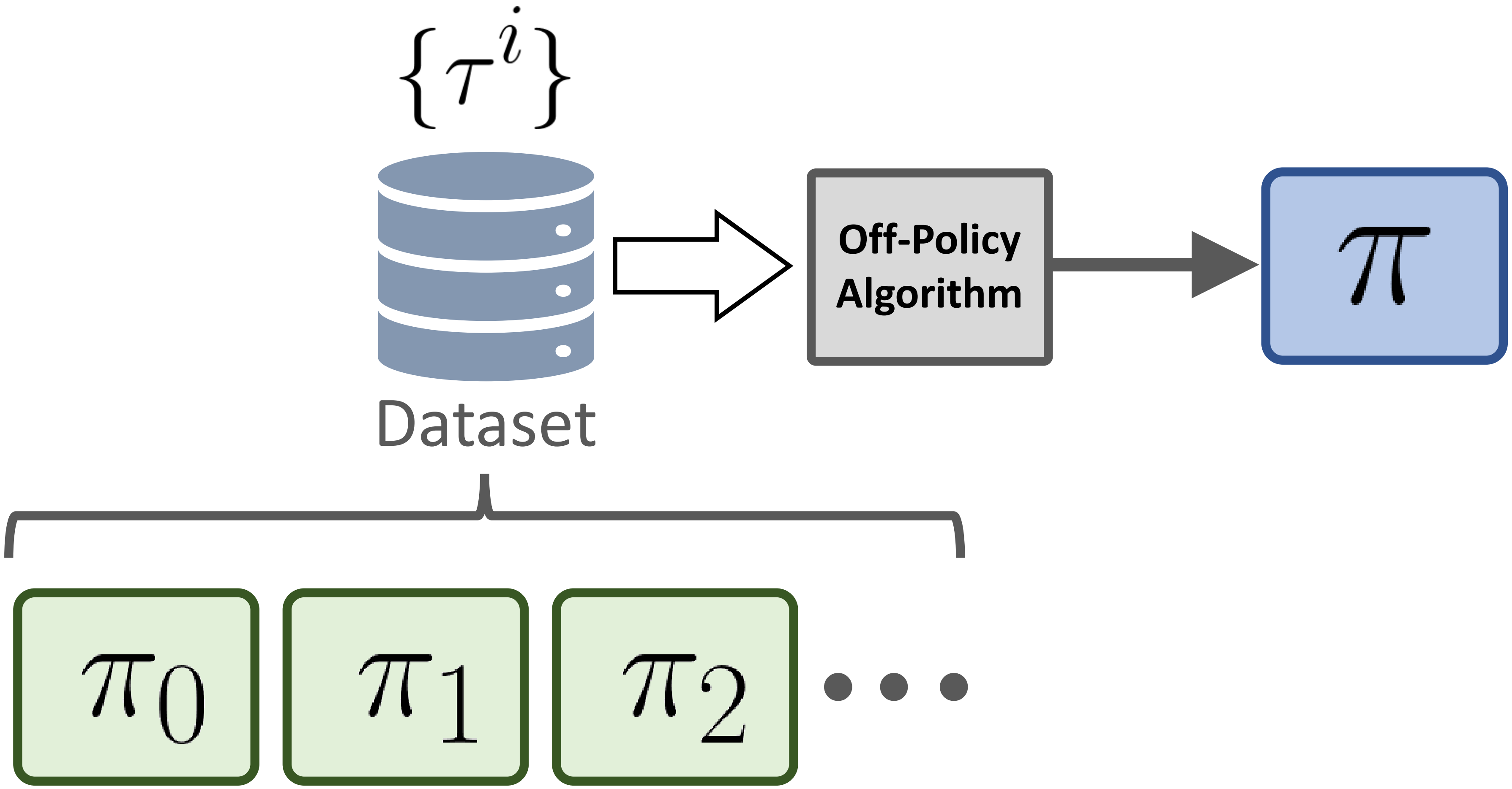


Off-Policy



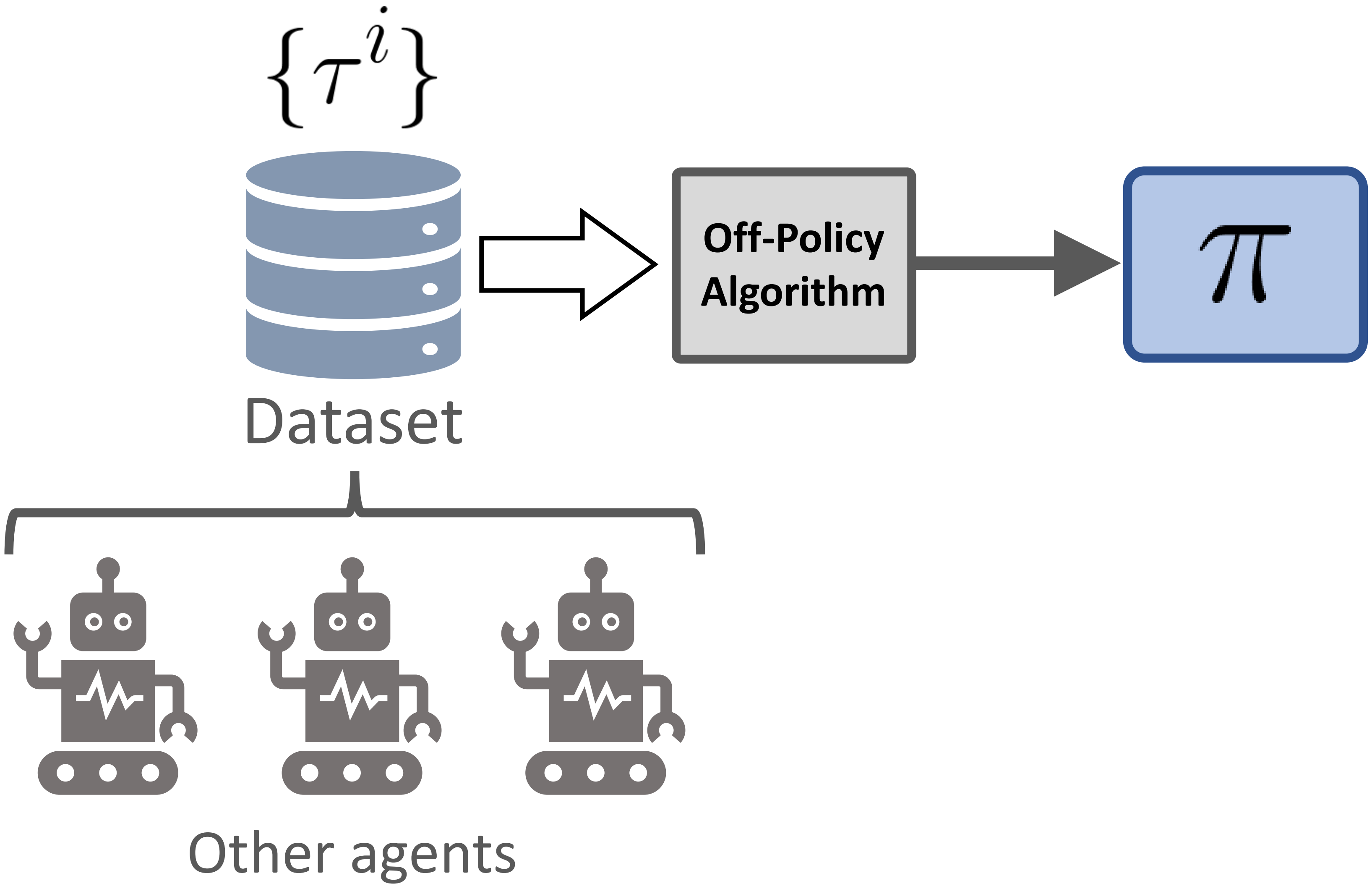
can come from anywhere

Off-Policy

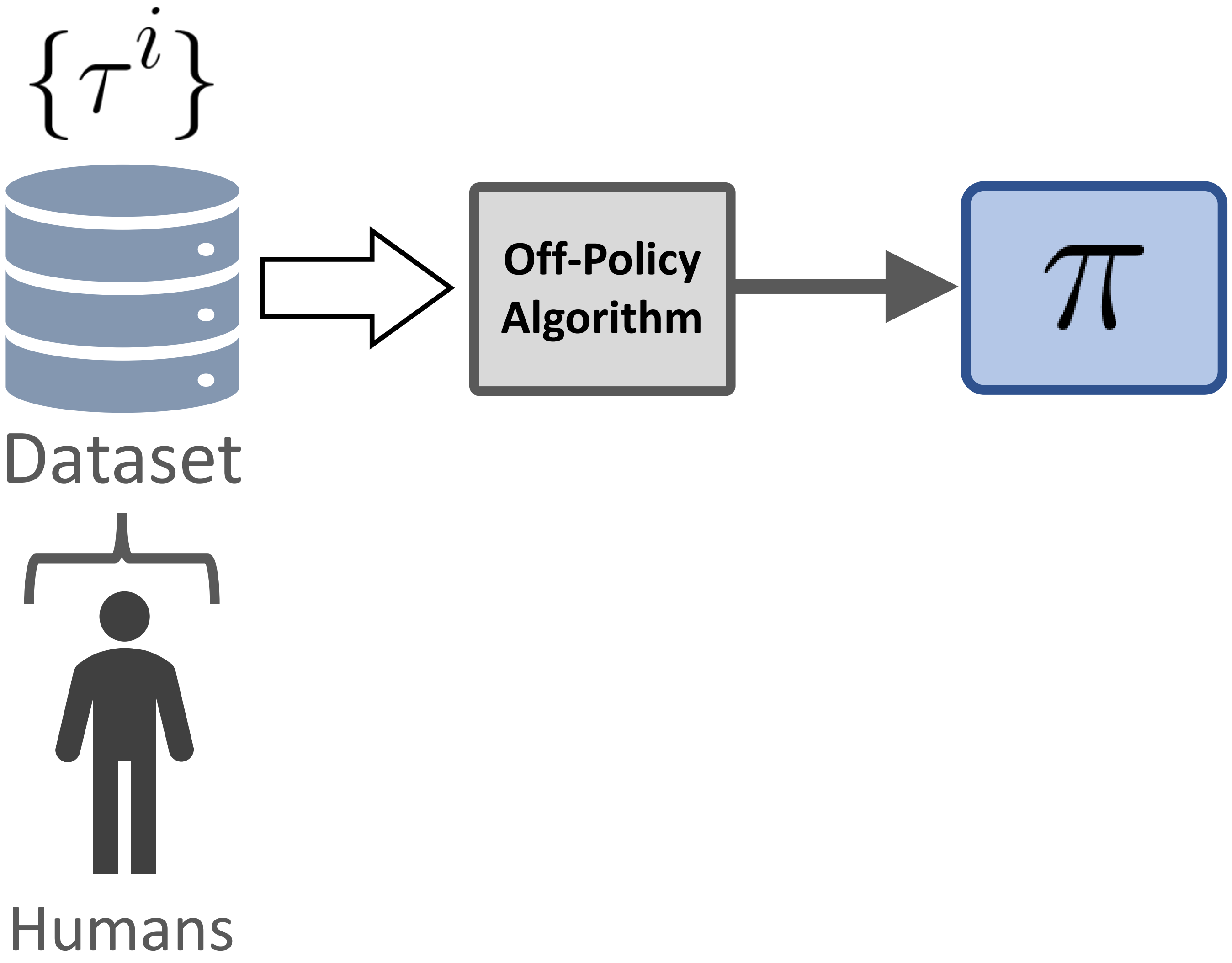


Policies from previous training iterations

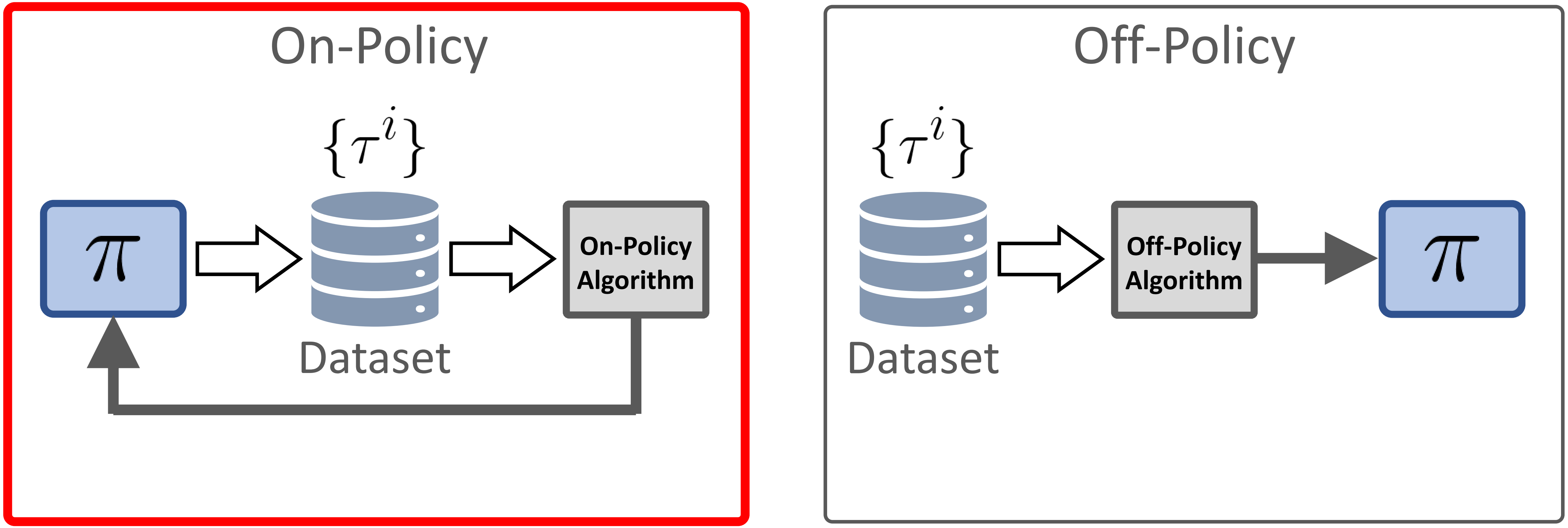
Off-Policy



Off-Policy



RL Algorithms



On-Policy (REINFORCE)

$$\nabla_\pi J(\pi) = \mathbb{E}_{\underline{\tau \sim p(\tau|\pi)}} \left[R(\tau) \sum_{t=0}^{T-1} \nabla_\pi \log \pi(\mathbf{a}_t | \mathbf{s}_t) \right]$$

Must be from
current policy

Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning
[Williams 1992]

On-Policy (REINFORCE)

ALGORITHM: REINFORCE

1: $\theta \leftarrow$ initialize policy parameters

2: **while** not done **do**

3: Sample trajectories $\{\tau^i\}$ from policy $\pi_\theta(\mathbf{a}|\mathbf{s})$

4: Estimate policy gradient

$\nabla_\theta J(\pi_\theta) \approx \frac{1}{N} \sum_i R(\tau^i) \sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)$

5: Update policy $\theta \leftarrow \theta + \alpha \nabla_\theta J(\pi_\theta)$

6: **end while**

7: return policy π_θ

Collect data with current policy

Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning [Williams 1992]

On-Policy (REINFORCE)

ALGORITHM: REINFORCE

1: $\theta \leftarrow$ initialize policy parameters

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6: **end while**

7: return policy π_θ

Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning [Williams 1992]

On-Policy (REINFORCE)

ALGORITHM: REINFORCE

```
θ ← initialize policy parameters

while not done do
    Sample trajectories {τ^i} from policy π_θ(a|s)
    Estimate policy gradient
    ∇_θ J(π_θ) ≈ (1/N) Σ_i R(τ^i) Σ_t ∇_θ log π_θ(a_t^i|s_t^i)
    Update policy θ ← θ + α ∇_θ J(π_θ)
end while

return policy π_θ
```

Perform just one grad update, then throw out data

Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning [Williams 1992]

On-Policy (Policy Gradient)

$$\nabla_\pi J(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\pi(\mathbf{s})}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[\nabla_\pi \log\pi(\mathbf{a}|\mathbf{s})\left(Q^\pi(\mathbf{s},\mathbf{a}) - V^\pi(\mathbf{s})\right)\right]$$

From current policy

Policy Gradient Methods for Reinforcement Learning with Function Approximation
[Sutton et al. 1999]

On-Policy (Policy Gradient)

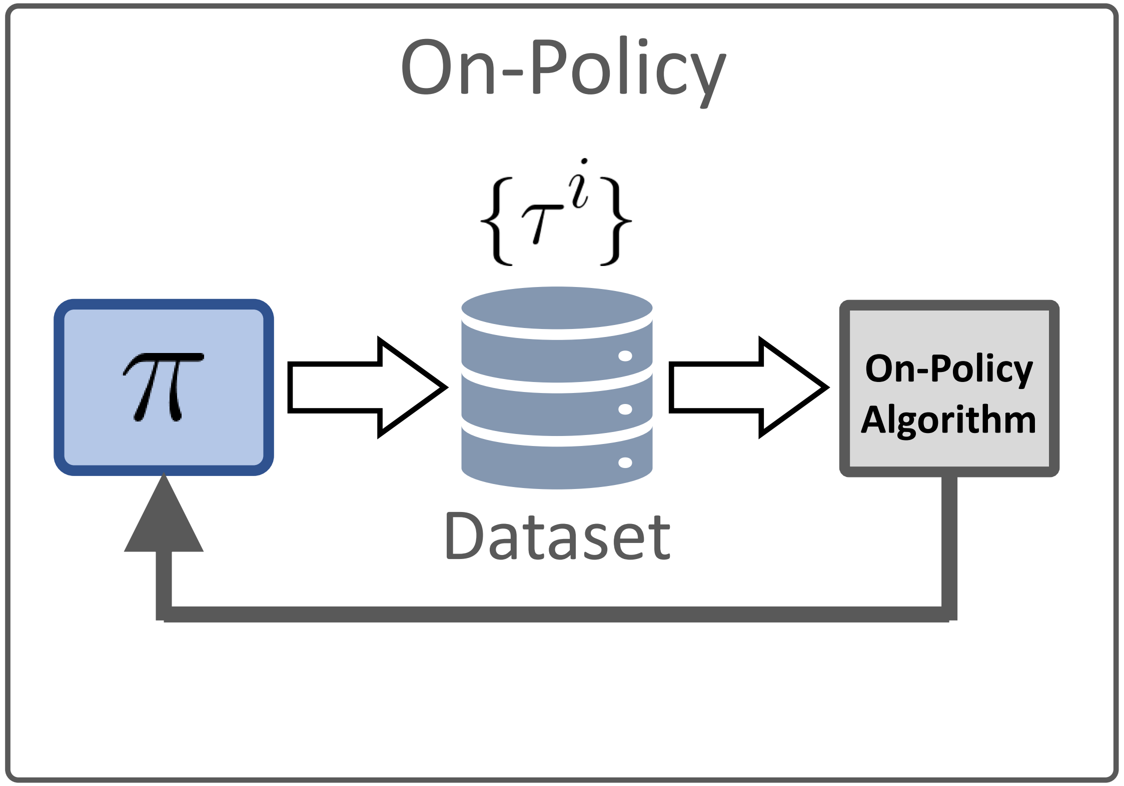
$$\nabla_\pi J(\pi) = \underbrace{\mathbb{E}_{\mathbf{s}\sim d_\pi(\mathbf{s})}}_{}\underbrace{\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}}_{}\left[\nabla_\pi \log \pi(\mathbf{a}|\mathbf{s})\left(Q^\pi(\mathbf{s},\mathbf{a}) - V^\pi(\mathbf{s})\right)\right]$$

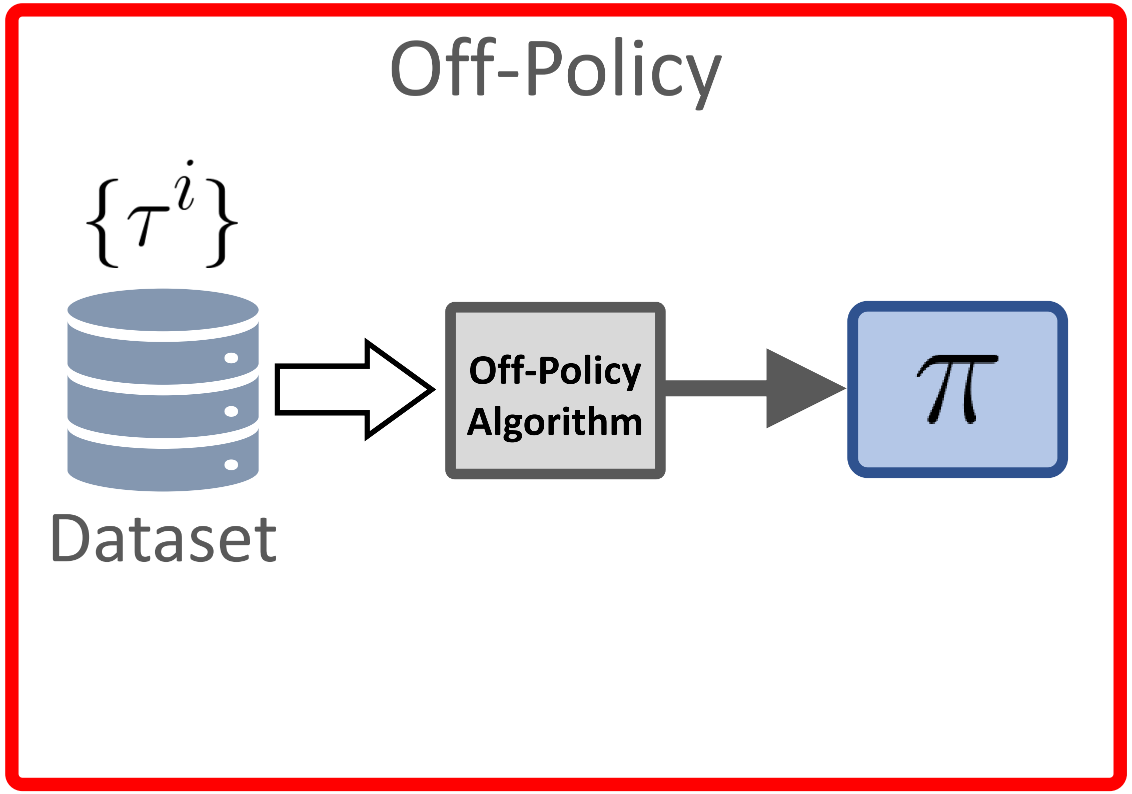
From current policy

- If data is not from π, PG methods can completely fail to learn anything

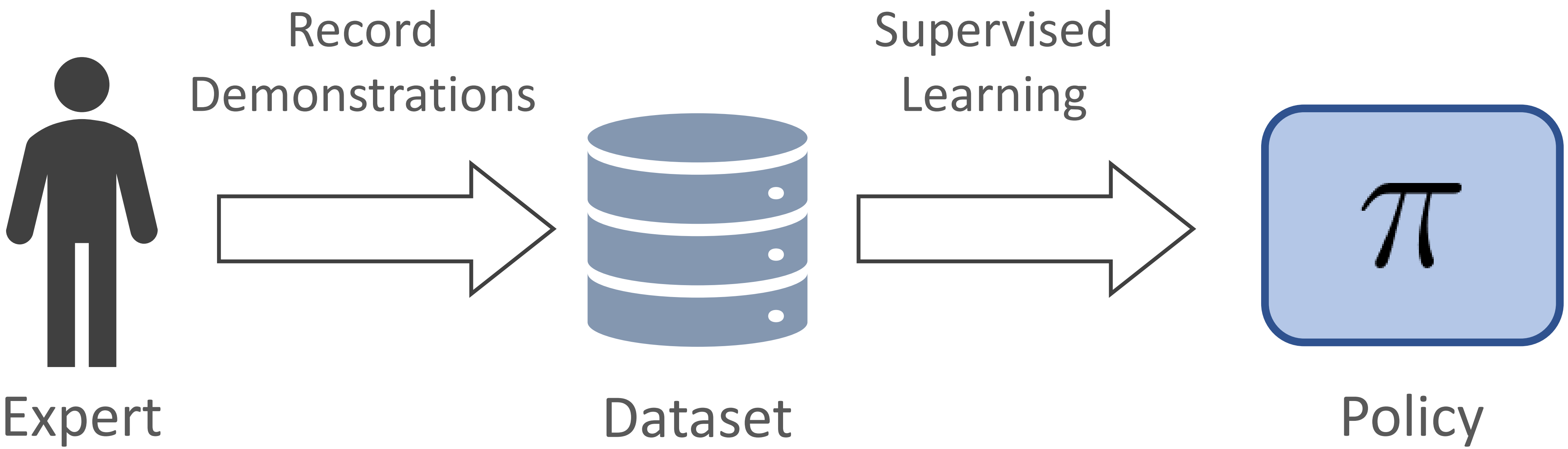
Policy Gradient Methods for Reinforcement Learning with Function Approximation [Sutton et al. 1999]

RL Algorithms





Behavioral Cloning

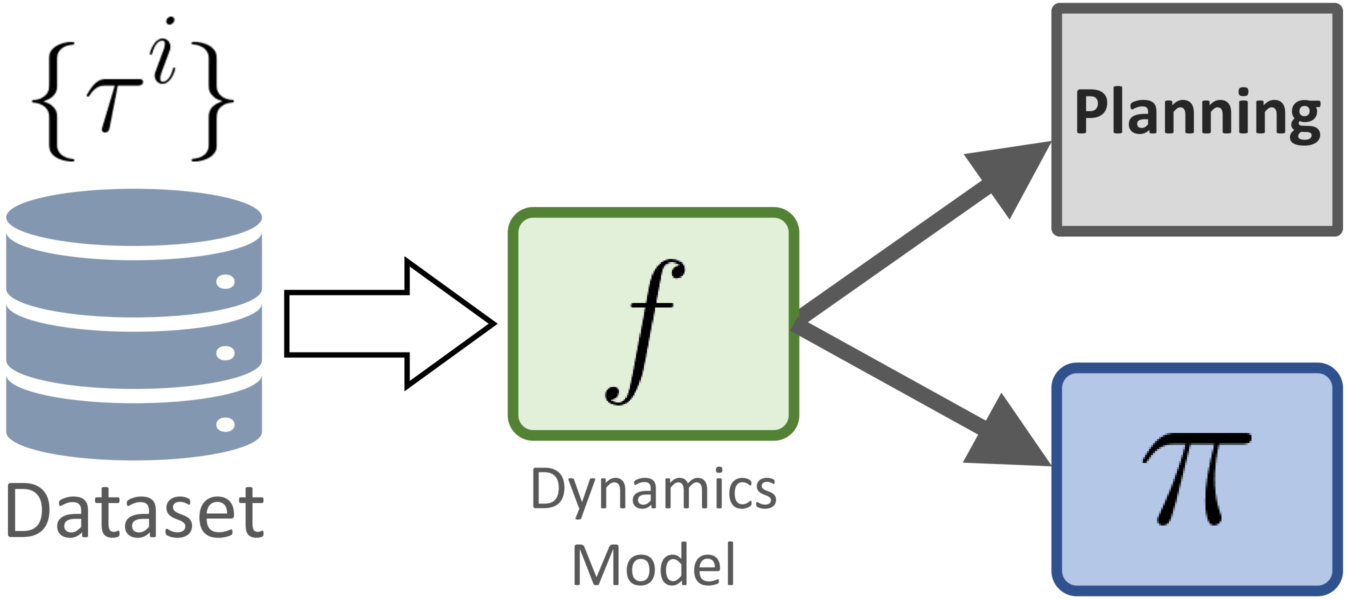


$$\min_{\pi} \; \mathbb{E}_{(\mathbf{o},\mathbf{a})\sim\mathcal{D}}\left[-\log \pi(\mathbf{a}|\mathbf{o})\right]$$

Off-Policy (Model-Based RL)

$$\arg\max_{f} \; \underline{\mathbb{E}_{(\mathbf{s},\mathbf{a},\mathbf{s}')\sim\mathcal{D}}} \left[\log f(\mathbf{s}'|\mathbf{s},\mathbf{a})\right]$$

- Dataset can come from anywhere, as long as it has sufficient coverage of states and actions.



Off-Policy (Q-Learning)

$$Q^{k+1} = \arg\min_{Q} \mathbb{E}_{(\mathbf{s},\mathbf{a},r,\mathbf{s}')\sim\mathcal{D}}\left[\left(\underbrace{\left(r + \gamma \max_{\mathbf{a}'} Q^k(\mathbf{s}',\mathbf{a}')\right)}_{\text{Current Q-function}} - Q(\mathbf{s},\mathbf{a})\right)^2\right]$$

Off-Policy (Q-Learning)

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Does not depend on Q^k!

- The data distribution can be an arbitrary distribution
- Tabular Q-Learning
  - If dataset covers *all* states and actions, then Q-learning will converge to the optimal Q-function
  - Can learn from a completely *random* policy, as long as agent observes every state and action at least once

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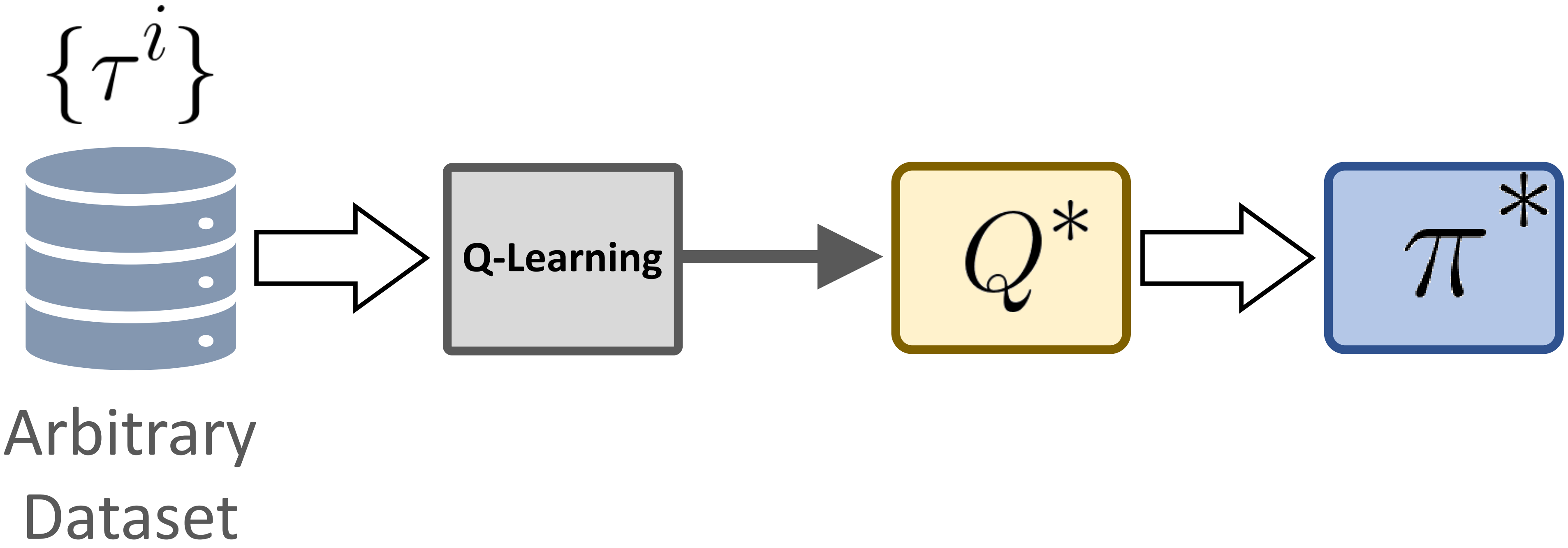

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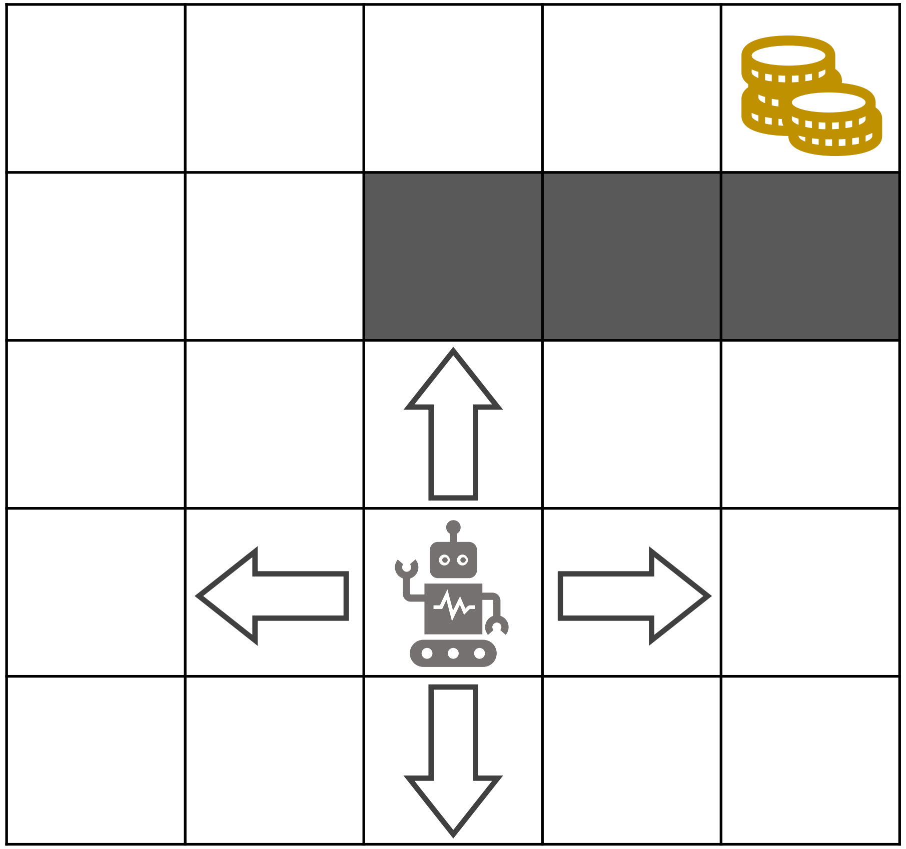

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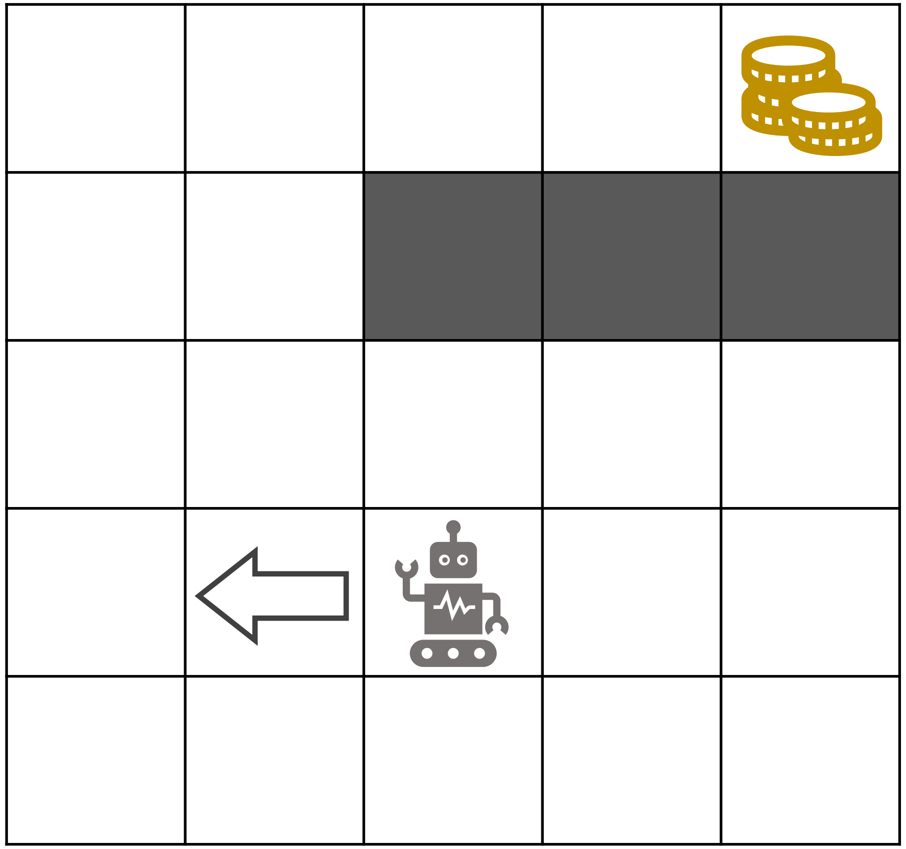

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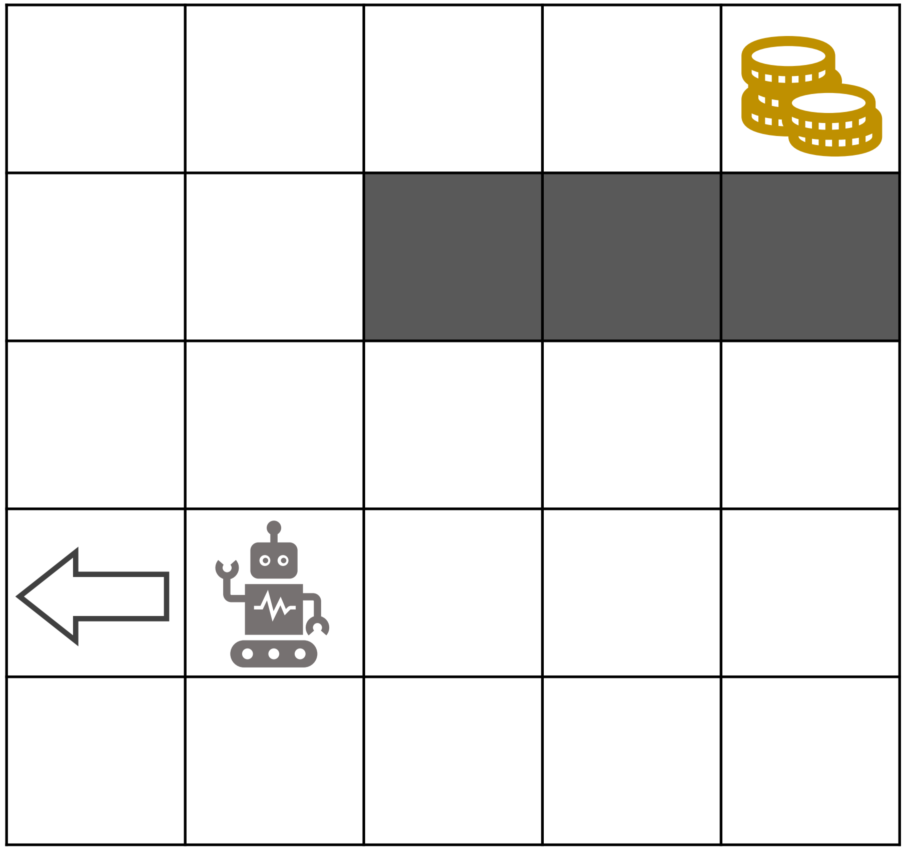

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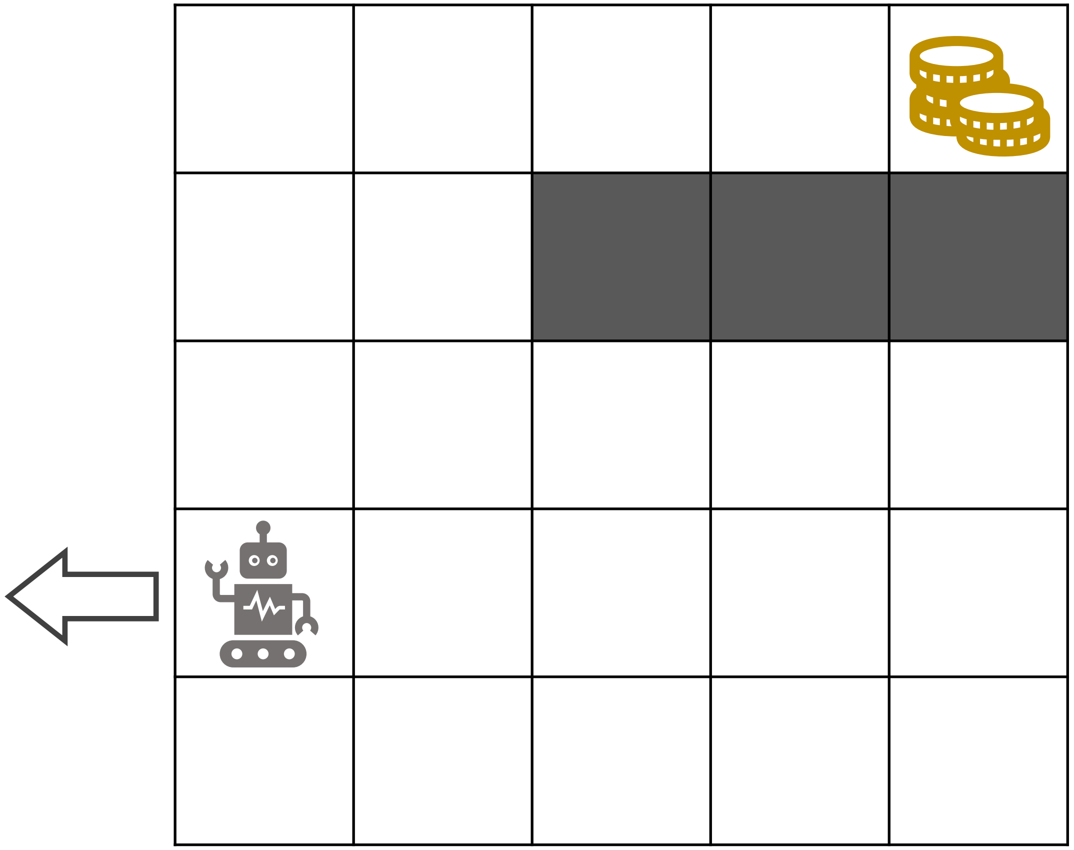
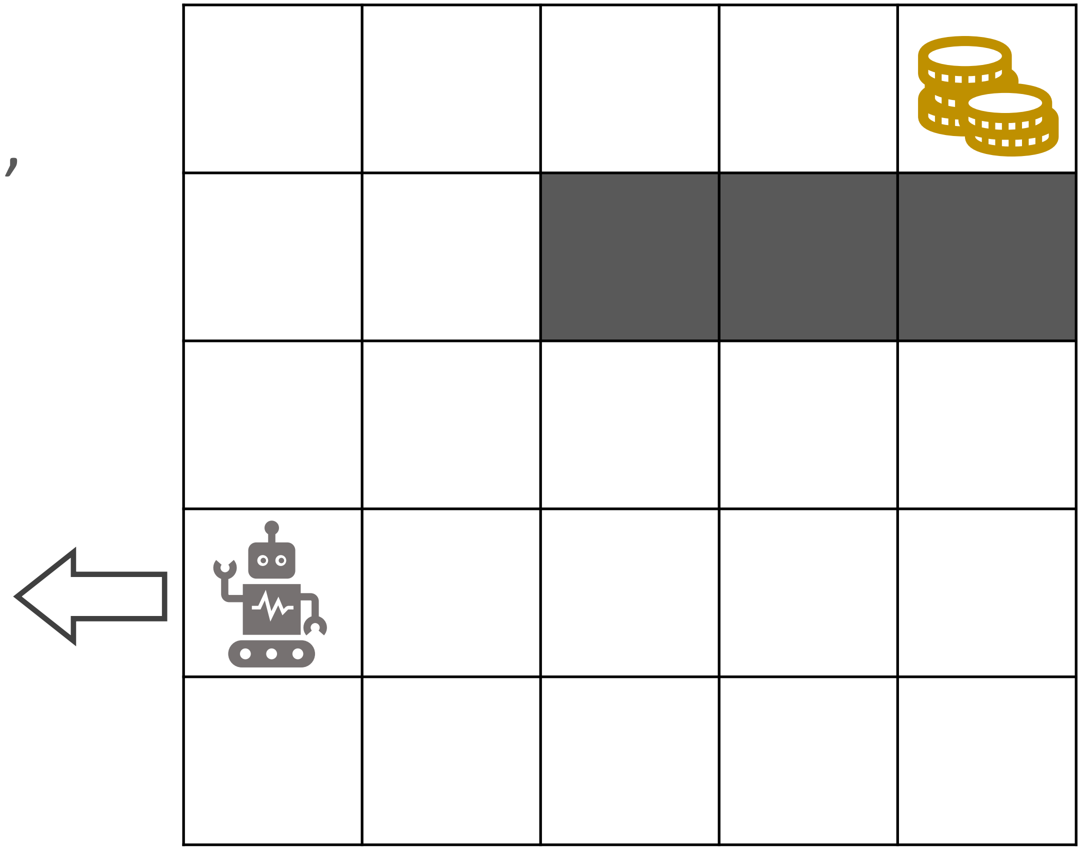


Off-Policy (Q-Learning)

$$Q^{k+1} = \arg\min_{Q} \mathbb{E}_{\underline{(\mathbf{s},\mathbf{a},r,\mathbf{s}')\sim\mathcal{D}}} \left[\left(\left(r + \gamma \max_{\mathbf{a}'} Q^k(\mathbf{s}', \mathbf{a}') \right) - Q(\mathbf{s}, \mathbf{a}) \right)^2 \right]$$

Independent of current model, but the characteristics of the data still matters.



Off-Policy (Q-Learning)

$$Q^{k+1} = \arg\min_{Q} \; \mathbb{E}_{(\mathbf{s},\mathbf{a},r,\mathbf{s}')\sim\mathcal{D}} \left[\left(\left(r + \gamma \max_{\mathbf{a}'} Q^k(\mathbf{s}',\mathbf{a}') \right) - Q(\mathbf{s},\mathbf{a}) \right)^2 \right]$$

- The data distribution can be an arbitrary distribution
- Q-Learning + function approximation
    - Not guaranteed to converge to the optimal Q-function
    - Can learn an effective policy from arbitrary dataset with sufficient coverage

Off-Policy (Q-Learning)

ALGORITHM: Q-Learning

$Q^0 \leftarrow$ initialize Q-function
$\mathcal{D} \leftarrow \{\emptyset\}$ initialize dataset

for iteration $k = 0, \ldots, n-1$ **do**
    Sample trajectory τ according to $Q^k(\mathbf{s}, \mathbf{a})$
    $\boxed{\text{Add transitions to dataset } \mathcal{D} = \mathcal{D} \cup \{(\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}_i')\}}$

    Calculate target values for each sample i:
    $y_i = r_i + \gamma \max_{\mathbf{a}'} Q^k(\mathbf{s}_i', \mathbf{a}')$

    Update Q-function:
    $Q^{k+1} = \arg\min_Q \ \mathbb{E}_{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{r}_i, \mathbf{s}_i') \sim \mathcal{D}} \left[(y_i - Q(\mathbf{s}_i, \mathbf{a}_i))^2 \right]$
end for

return Q^n

Keep data from previous iterations for better coverage

Off-Policy (SAC)

$$\hat{J}(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\pi(\mathbf{s})}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[\underline{\hat{Q}^\pi}(\mathbf{s}, \mathbf{a})\right]$$

Off-Policy (SAC)

$$\hat{J}(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\pi(\mathbf{s})}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[\hat{Q}^\pi(\mathbf{s},\mathbf{a})\right]$$

$$\nabla_\pi \hat{J}(\pi) \approx \nabla_\pi J(\pi)$$

Off-Policy (SAC)

$$\hat{J}(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\pi(\mathbf{s})}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[\hat{Q}^\pi(\mathbf{s},\mathbf{a})\right]$$

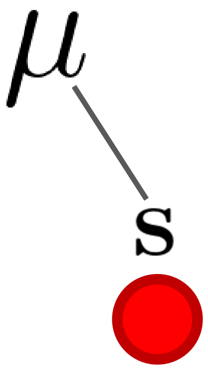
$$\Downarrow$$

$$\hat{J}(\pi) = \mathbb{E}_{\mathbf{s}\sim \boxed{d_\mu(\mathbf{s})}}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[\hat{Q}^\pi(\mathbf{s},\mathbf{a})\right]$$

$\mu(\mathbf{a}|\mathbf{s})$: behavior policy

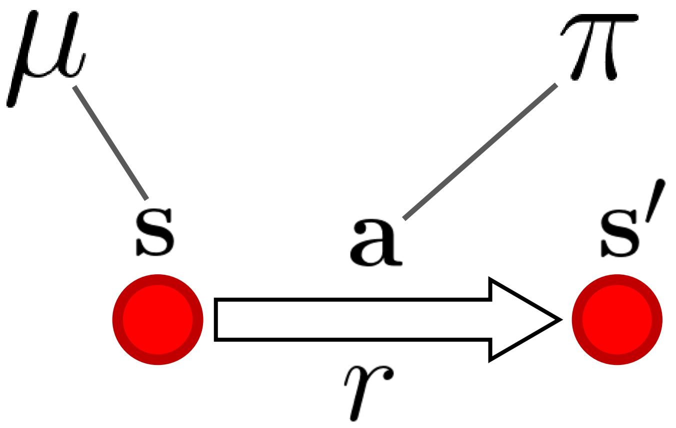
Surrogate Objective

$$\hat{J}(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\mu(\mathbf{s})}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[Q^\pi(\mathbf{s},\mathbf{a})\right]$$



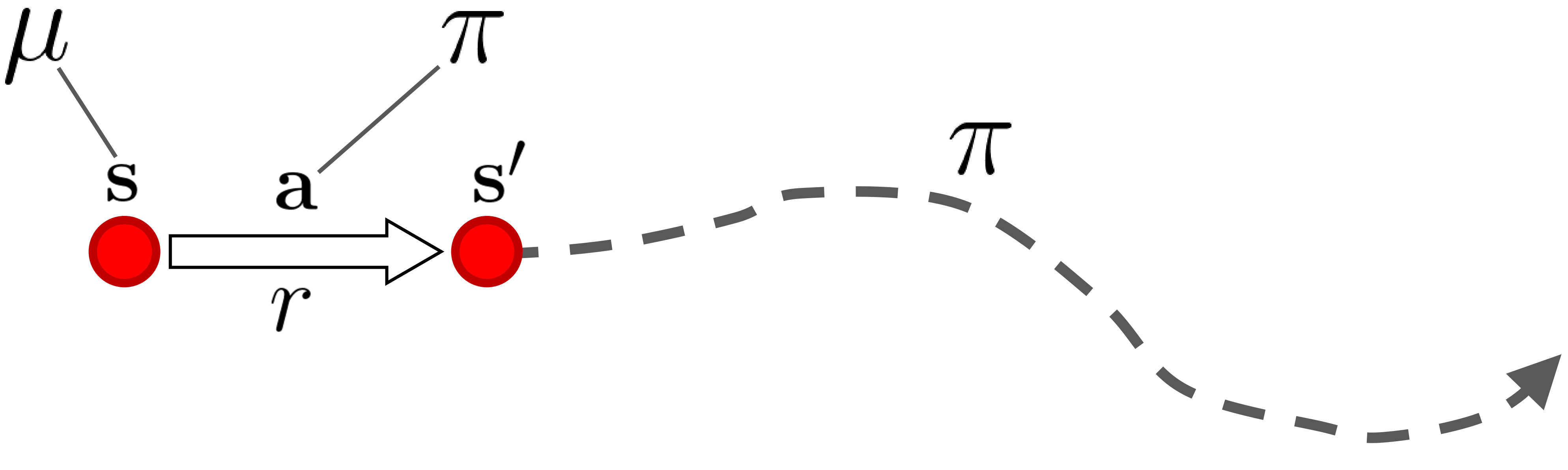
Surrogate Objective

$$\hat{J}(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\mu(\mathbf{s})} \underline{\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}} \left[Q^\pi(\mathbf{s},\mathbf{a})\right]$$



Surrogate Objective

$$\hat{J}(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\mu(\mathbf{s})}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[Q^\pi(\mathbf{s},\mathbf{a})\right]$$



π is trying to maximize return starting in states visited by μ

Off-Policy (SAC)

$$\hat{J}(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\mu(\mathbf{s})}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[Q^\pi(\mathbf{s},\mathbf{a})\right]$$

- If $\mu(\mathbf{a}|\mathbf{s}) = \pi^*(\mathbf{a}|\mathbf{s})$, then $\pi \to \pi^*$
- If $\mu(\mathbf{a}|\mathbf{s}) \neq \pi^*(\mathbf{a}|\mathbf{s})$, then algorithm will still learn *some* policy, but it might not be optimal
- Will not work for a completely random behavior policy, even if it covers all states and actions

Off-Policy (SAC)

ALGORITHM: SAC

$Q^0 \leftarrow$ initialize Q-function
$\pi^0 \leftarrow$ initialize policy
$\mathcal{D} \leftarrow \{\emptyset\}$ initialize dataset

for iteration $k = 0, ..., n-1$ **do**
  Sample trajectory τ according to $\pi^k(\mathbf{a}|\mathbf{s})$
  Add transitions to dataset $\mathcal{D} = \mathcal{D} \cup \{(\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i)\}$

  Calculate target values for each sample i:
  $y_i = r_i + \gamma \mathbb{E}_{\mathbf{a}' \sim \pi^k(\mathbf{a}'|\mathbf{s}'_i)} \left[Q^k(\mathbf{s}'_i, \mathbf{a}') \right]$

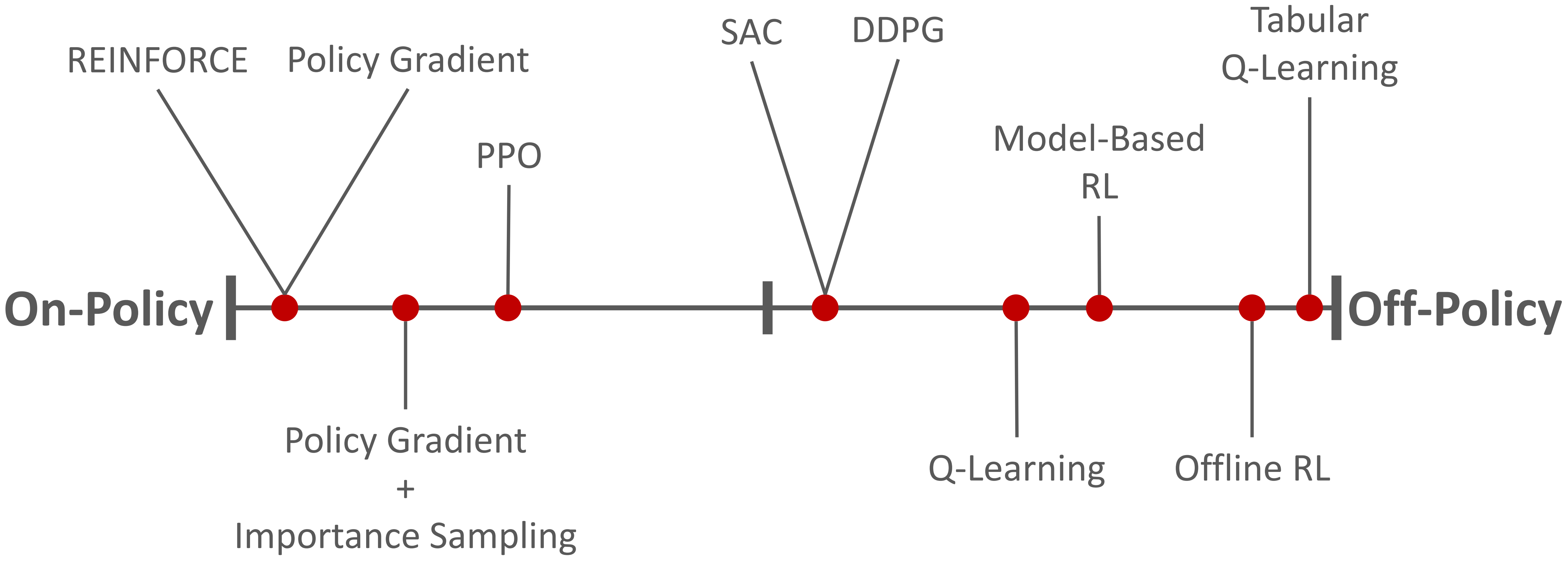
  Update Q-function:
  $Q^{k+1} = \arg\min_Q \ \mathbb{E}_{(\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i) \sim \mathcal{D}} \left[(y_i - Q(\mathbf{s}_i, \mathbf{a}_i))^2 \right]$

  Update policy:
  $\pi^{k+1} = \arg\max_\pi \ \mathbb{E}_{\mathbf{s}_i \sim \mathcal{D}} \mathbb{E}_{\mathbf{a} \sim \pi(\mathbf{a}|\mathbf{s}_i)} \left[Q^{k+1}(\mathbf{s}_i, \mathbf{a})) \right]$
end for

return π^n

π improves every iteration, and gets closer to π^*

On-Policy vs Off-Policy



Trade-Offs

On-Policy	Off-Policy
✗ Sample inefficient	✓ Sample efficient
✓ Fast wall-clock time	✗ Slow wall-lock time
✓ Typically better asymptotic performance	✗ Typically worse asymptotic performance
✓ More stable and easy to tune	✗ More unstable and hard to tune
✗ Exploration limited by action distribution	✓ Flexible exploration

On-Policy Exploration

Exploration: what actions can the policy take?

$$\nabla_\pi J(\pi) = \mathbb{E}_{\mathbf{s} \sim d_\pi(\mathbf{s})} \mathbb{E}_{\mathbf{a} \sim \pi(\mathbf{a}|\mathbf{s})} \left[\nabla_\pi \log \pi(\mathbf{a}|\mathbf{s}) \left(Q^\pi(\mathbf{s}, \mathbf{a}) - V^\pi(\mathbf{s}) \right) \right]$$

On-Policy Exploration

Exploration: what actions can the policy take?

$$\nabla_\pi J(\pi) = \mathbb{E}_{\mathbf{s}\sim d_\pi(\mathbf{s})}\mathbb{E}_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}\left[\underline{\nabla_\pi \log \pi(\mathbf{a}|\mathbf{s})}\left(Q^\pi(\mathbf{s},\mathbf{a}) - V^\pi(\mathbf{s})\right)\right]$$

Policy Gradient:

- Action distribution must have a differentiable log-likelihood
- Limited to simple action distributions with easy to compute log-likelihoods

Off-Policy Exploration

Exploration: what actions can the policy take?

$$\hat{J}(\pi) = \underline{\mathbb{E}_{\mathbf{s} \sim d_\mu(\mathbf{s})}} \mathbb{E}_{\mathbf{a} \sim \pi(\mathbf{a}|\mathbf{s})} \left[Q^\mu(\mathbf{s}, \mathbf{a}) \right]$$

SAC:

- Do not need to differentiate behavior policy μ
- Can collect data using any action distribution as long as it has good coverage of actions
- E.g. temporally correlated actions, epsilon-greedy, multi-modal distributions, mixture models, etc.

Summary

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- Off-Policy Algorithms
- Trade-Offs